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THE CALCULATIONS OF TWO COMPRESSIBILITY MECHANISMS IN EQUATIONS OF SOIL DYNAMICS

G. M. Lyakhov, et al

Foreign Technology Division
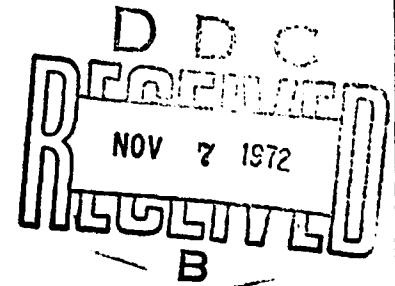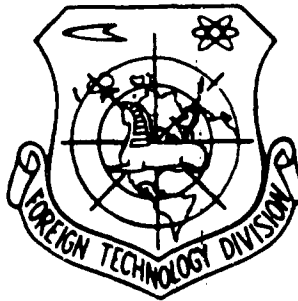Wright-Patterson Air Force Base, Ohio

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FOREIGN TECHNOLOGY DIVISION

THE CALCULATIONS OF TWO COMPRESSIBILITY
MECHANISMS IN EQUATIONS OF SOIL DYNAMICS

by

G. M. Lyakhov and N. I. Polyakova

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A system of equations is proposed which determines the unidirectional movement of the soil with the simultaneous calculation of its compressibility as a three-component medium and also as the compressibility of the soil skeleton. The soil characteristics and the values of the pressures at which it is necessary to calculate both compressibility mechanisms were determined experimentally.

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Soil Structure						
Soil Property						

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THE CALCULATIONS OF TWO COMPRESSIBILITY
MECHANISMS IN EQUATIONS OF SOIL DYNAMICS

G. M. Lyakhov and N. I. Polyakova

(Moscow)

A system of equations is proposed which
determines the unidirectional movement of the
soil with the simultaneous calculation of its
compressibility as a three-component medium
and also as the compressibility of the soil
skeleton. The soil characteristics and the
values of the pressures at which it is neces-
sary to calculate both compressibility mechan-
isms were determined experimentally.

1. Soil is an aggregate of solid mineral particles (grains)
which form its skeleton, and pore spaces between these particles
which are filled with water and air. During the action of a
dynamic load deformation of the skeleton occurs along with the
simultaneous compression of the water, air and solid particles.
In accordance with this two mechanisms of compressibility and
two mechanisms of stresses need to be examined [1, 3].

The first mechanism corresponds to the compressibility of the
soil skeleton. The deformation is associated with the destruction
of the bonds between the solid particles and their displacement
relative to one another. Experiments indicate that there is stress
in soils as well as in solids, and depending on the orientation
the state of stress is determined by the tensor value.

The second mechanism corresponds to the compressibility of the soil as a three-component medium, i.e., a mixture of solid particles, water and air. The deformation of the soil is connected with the volume deformation of gaseous, liquid and solid particles. From experiments it follows that the state of stress, just as in liquids, is determined by the scalar gravity — the pressure.

During the dynamic processes one of the stresses frequently turns out to be substantially smaller and it can be ignored. At a stress where the volume deformation of the soil ε is substantially less than the allowance α_1 for the volume of air in the soil, the compressibility is determined generally by the compressibility of the skeleton. When $\varepsilon > \alpha_1$, the compressibility of the soil is determined generally by its compressibility as three-component medium. When ε approaches α_1, it is necessary to take into account both compressibility mechanisms. For example, in water saturated soil with $\alpha_1 = 0.01$ under a load of several atmospheres and in unsaturated soil with $\alpha_1 = 0.3$ under loads of several tens of atmospheres.

Let us find the equations which determine the soil movement taking into account both compressibility mechanisms. Let us first examine the soil properties as a three-component medium. During short-term dynamic processes the value of the components in each of the soil particles according to the mass remains unchanged, but by volume is changed. Let us designate α_1, α_2, α_3, as the value by volume, ρ_1, ρ_2, ρ_3, — density, and c_1, c_2, c_3, — the speed of sound, respectively, in the gaseous, liquid and solid components at atmospheric pressure.

Then, the soil density ρ_0 and bulk density (volume weight) of the skeleton γ_0 can be determined by the expressions:

$$\rho_0 = \alpha_1 \rho_1 + \alpha_2 \rho_2 + \alpha_3 \rho_3, \quad \gamma_0 = \alpha_3 \rho_3. \qquad (1.1)$$

In this case

$$a_1 + a_2 + a_3 = 1.$$

Based on the assumption that with applied stress p the pressure on all the components is also equal to p, and that the density of each component can be determined by law, corresponding to its compression in the free state, the equation of the state of the soil [1] as a three-component medium was obtained:

$$p = \rho_0 \left\{ a_1 \left(\frac{p}{p_0} \right)^{-\frac{1}{\kappa_1}} + a_2 \left[\frac{\kappa_2 (p - p_0)}{\rho_2 C_2^2} + \right. \right. \qquad \left. \right\}^{-\frac{1}{\kappa_2}} +$$
$$+ a_3 \left[\frac{\kappa_3 (p - p_0)}{\rho_3 C_3^2} + 1 \right]^{-\frac{1}{\kappa_3}} \right\}^{-1}, \qquad (1.2)$$

where κ_1, κ_2, κ_3 — indices of the isentropes of the air, water and quartz, accepted as equal to 1; 4; 3.

The unloading of the soil as a mixture proceeds according to this same equation. In Fig. 1 the graph of the relationship p = = p(ε), corresponding to (1, 2), is represented by curve 1. Deformation ε is associated with the density of the soil ρ and the specific volume V by the expression

$$\varepsilon = \frac{\rho_0 - \rho}{\rho} = \frac{V - V_0}{V_0}. \qquad (1.3)$$

Let us examine the properties of the soil as a solid. It is assumed that in the case of one-dimensional planes and spherical movements, the soil properties are determined by two relationships known from the experiment — the law of cubic compression and the condition of plasticity

$$\sigma = \sigma_1 + 2 \sigma_2 = \psi (\rho, \rho_m), \qquad (1.4)$$
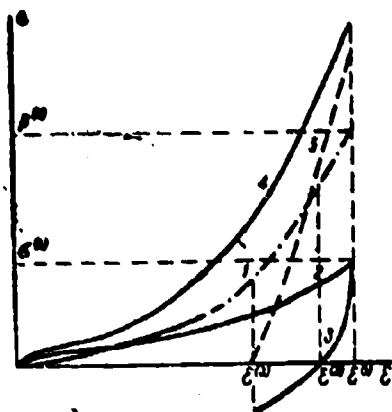$$f (\sigma_1, \sigma_2) = 0. \qquad (1.5)$$

Fig. 1.

Here σ_1 and σ_2 — the stresses acting in the direction of the application of the external load as well as in a perpendicular direction, σ — average normal stress, ρ — current stress, and ρ_m — previously attained maximum soil density.

In the first approximation with stresses not too great

$$\sigma_2 = \kappa_\tau \, \sigma\,(\sigma_1)\, \sigma_1. \qquad (1.6)$$

In many soils $\kappa_\tau = $ const. From here (1.4) it can be expressed in the form

$$\sigma_1 \,(1 + 2\,\kappa_\tau) = \psi\,(\rho,\ \rho_m)$$

or

$$\sigma_1 = \varphi\,(\varepsilon_1,\ \varepsilon_m). \qquad (1.7)$$

when ε_m — the maximum deformation previously proven by the soil.

Let us examine the movement of the medium based on the compressibility mechanisms. During quick-acting processes the water and air are not expelled from the soil; they appear to be forced into the pores. Therefore, it can be assumed that each particle

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of the soil making up all three components, acts as an individual unit with a velocity $u = u(x, t)$ coinciding with the direction of the external load (x — spatial coordinate, t — time). The movement of the medium is therefore associated with the overall stress in the soil P, equal to the sum of pressures p in the soil as a mixture, and the stresses in the skeleton σ_1, acting in the direction of the movement. From here it follows that the unidimensional movement is determined by a system of equations:

$$\frac{d\rho}{dt} + \rho\,\frac{\partial u}{\partial x} + \nu\,\frac{u\rho}{x} = 0, \quad \rho\,\frac{du}{dt} +$$
$$+ \frac{\partial}{\partial x}\,(p - \sigma_1) - \nu\,\frac{\sigma_1 - \sigma_2}{x} = 0. \qquad (1.8)$$

With planar and spherical symmetry, ν is equal to 0 and 2, respectively. Equations (1.8), (1.2) in conjunction with (1.7) and (1.5) or with (1.4) and (1.5)) form a closed system of five equations with five functions: σ_1, σ_2, p, ρ, u. This system pertains to the increment cycle of the external load, i.e., to the period of increase in the deformation.

During unloading the soil density as a mixture is lowered according to the law of loading, and the deformations in the skeleton change according to a law different from that of the law of loading. During unloading $\sigma_1 = \sigma_1(\varepsilon,\ \varepsilon)^{(1)}$, where $\varepsilon^{(1)}$ — the maximum deformation attained during loading. In this case

$$\varepsilon^{(1)} > \varepsilon_m, \quad \sigma_1\,(\varepsilon,\ \varepsilon_m) > \sigma_1\,(\varepsilon,\ \varepsilon^{(1)}). \qquad (1.9)$$

The dependences $\sigma_1(\varepsilon)$ during loading and unloading are represented by curves 2 and 3 in Fig. 1. Curves 4 and 5 pertain to the sum $P = p - \sigma_1$, during loading and unloading, respectively.

Unloading in various soils can proceed differently. In soils where the rate of water loss is small, the water and air just as

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during loading remains in the restrained state. Each soil particle expands as a single unit, and maintains the common rate of all the components. The movement is determined by this same system of equations also by the fact that during loading only the stress in the skeleton is determined by the equation $\sigma_1 = \dot{\sigma}_1(\varepsilon, \varepsilon^{(1)})$.

With a reduction in the external load when $\varepsilon = \varepsilon^{(2)}$ (Fig. 1) the stress in the skeleton falls to zero. All of the load in this case is taken only by the soil as a mixture. When $\varepsilon < \varepsilon^{(2)}$ the expansion of the soil as a mixture causes the formation of tensile stresses in the skeleton. In this case

$$p > |\sigma_1|.$$

The system of equations which determines the movement of the soil, in this case, is valid. With a reduction of the external load, the tensile stresses increase, and the deformation is reduced. At the moment the external load drops to zero, when the deformation of the soil is equal to $\varepsilon^{(3)}$ (Fig. 1), pressure p is positive, and the stress σ_1 is negative. In this case

$$p = |\sigma_1|,$$

$\varepsilon^{(3)}$ corresponds to the residual deformation of the soil. The greater the content the water in the pores, the less intense the expansion of the soil $\varepsilon^{(3)}$.

Subsequently, after the termination of the action of the dynamic load, the water and air gradually begin to move to the side of the free surface, and the pressure in the soil as a mixture falls to zero. This process is not directly connected with the propagation of the wave. In soils with a high rate of water loss during unloading when $\varepsilon < \varepsilon^{(2)}$, displacement of the liquid and gaseous components can take place relative to the skeleton. In this case the above examined system of equations is no longer satisfied.

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Certain water saturated sands pass into the state of lique-
faction during the propagation of blast waves. This associate
with, apparently, the arise of tensile stresses in the soil after
the passage of the shock wave front. Similar reasoning was offered
by V. I. Belokopytov.

From the experiment [3] it is known that with a reduction of
the content of air α_1 in the water saturated sand, the probability
of the onset of liquefaction is increased. The following explana-
tion can be given for this. With a decrease in α_1 curve 1 in Fig. 1
approaches the ordinate axis. Therefore, during similar external
loads in a sand with a smaller α_1, the values of $p^{(1)}$ will be
larger and $\sigma^{(1)}$ smaller. The maximum possible value of the tensile
forces increases with an increase in $p^{(1)}$. In this way with a
decrease in α_1 the value of $p^{(1)}$ and consequently the probability
of liquefaction increase.

After liquefaction sets in, the movement of the soil is deter-
mined by the equations (1.2) and (1.8), in which $\sigma_1 = \sigma_2 = 0$. The
velocity of the shock wave front D and the speed of sound c in the
calculation of the two compressibility mechanisms are determined
by the equations:

$$D = \sqrt{\frac{\rho}{\rho_0}\frac{p - \sigma_1 - p_0 + \sigma_0}{\rho - \rho_0}} ; \qquad c = \sqrt{\frac{d(p - \sigma_1)}{d\rho}} \qquad (1.10)$$

where ρ is associated with p by equation (1.2).

2. Let us examine the results of the experiments for the
determination of the dynamic compressibility of the soil and the
conditions under which it is necessary to calculate the two com-
pressibility mechanisms. The experiments were conducted on a
sandy soil in a water saturated and unsaturated condition. The

soil was placed in a cylindrical tank with cement walls and bot.
(the diameter of the tank was 2 m, the height 1.85 m).

The volume weight (bulk density) of the skeleton γ_0 =
= 1.50-1.56 g/cm^3. The amount of air by volume in the unsaturated
soil α_1 = 0.25-0.37 (moisture W = 4-10%), and in the water saturated
soil α_1 = 0.02-0.04 (W = 23-27%). The grain-zize distribution is
given below:

Table

δ, мм	> 1	1—0,5	0,5—0,25	0,25 0,1	0,1—0,01	< 0,01
β, %	:0—15	15—1ъ	30—40	15—25	5—8	2—4

Here, δ — diameter of the particles in mm, β — their content in
percent.

Waves were produced by a blast of a slab charge placed on the
surface of the soil at the top of the tank. From the surface the
charge disturbed a layer of soil having a thickness

$$h = \varkappa C;$$

C — density of the charge (weight in kg per 1 m^2),

$$\varkappa = 3 \; м^3/кг.$$

The parameters of the waves were recorded with the aid of
strain gages whose readings were recorded on loop oscillographs.

The experiments indicated that the wave is a shock wave when
the maximum pressure is greater than p_s. In water saturated and
unsaturated soils p_s is equal to 7-10 and 4-6 kg/cm^2.

In Fig. 2 graphs of the dependence of the wave front veloci
D and the maximum pressure D_m on the relative distance

$$R^\circ = \frac{R}{C} \frac{\varkappa^3}{\varkappa_C},$$
(2.1)

where R — distance from the site of the (slab) charge, m; C —
density of the slab charge, kg/m^2; Curves 1, 2 pertain to the water
saturated soil, and 3-4 — to the unsaturated soil. When the wave
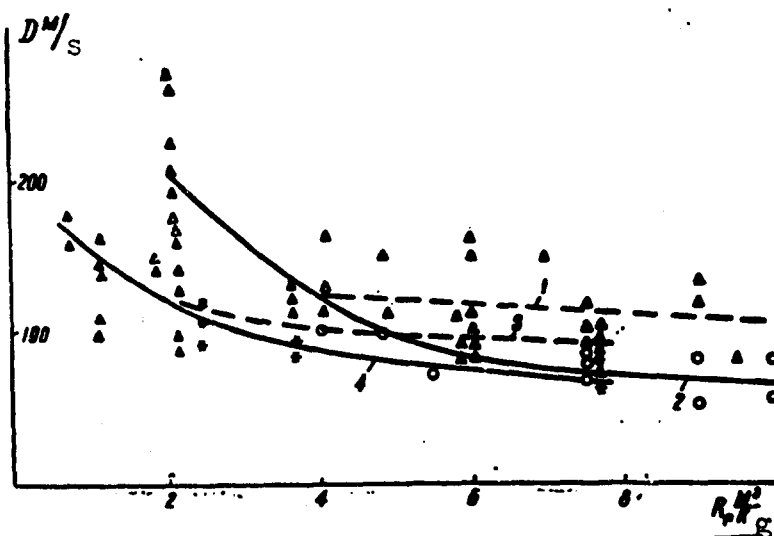is a shock wave, D and D_m coincide.

Fig. 2.

From the graphs it is obvious that at close distances to the
slab charge, the velocities in the water saturated soil is sub-
stantially higher than in the unsaturated soil. With an increase
in R° the differences diminish.

Figure 3 portrays graphs of the dependence of the maximum
pressure P_1 on R° in water saturated soil (curve 1) and in un-
saturated soils (curve 2). The scattering of the experimental
points is great which is associated with the fluctuations of
α_1 and γ_0 of the first order in the various experiments. However,
all the points pertaining to water saturated soil, are appreciably
higher. The maximum pressure in water saturated soil is reduced

significantly more slowly with distance. The found dependence of the maximum pressure on the distance can be presented in the form

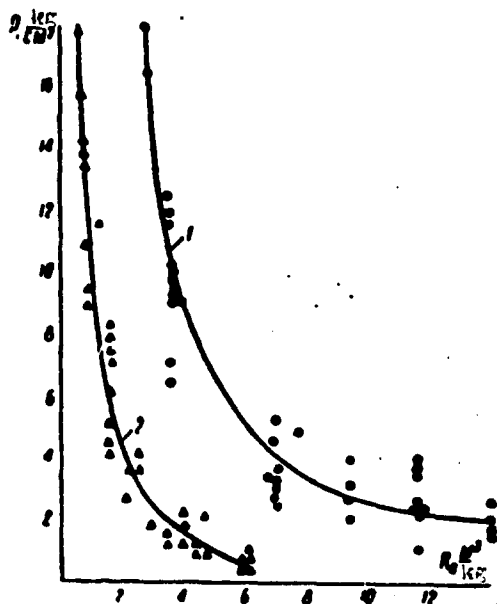$$P_1 = \kappa \left(\frac{C}{R} \right)^\mu.$$

(2.2)

Fig. 3.

For water saturated sand $\kappa = 55$, $\mu = 1.3$, and for unsaturated sand $\kappa = 12$, $\mu = 1.5$.

Let us construct the relationship $P_1 = P_1(\varepsilon)$ according to the experimental data of P_1 and D in the range of pressures where the wave is a shock wave. At the shock wave front the relationship is satisfied

$$D = \sqrt{\frac{\rho}{\rho_0} \frac{P_1}{\rho_m - \rho_0}} \quad \text{or} \quad \varepsilon = \frac{P_1}{\rho_0 D^2}.$$

(2.3)

Here, P_0 is taken to be equal to zero.

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Graphs $P_1 = P_1(\epsilon)$ in Fig. 4 correspond to the dynamic compression at the shock wave front of a water saturated (curve 1) and unsaturated (curve 2) soil. They were plotted in accordance with the equation (2.3) based on the experimental curve in Figs. 2 and 3. Curves 3-8 express the relationship $p = p(\epsilon)$, calculated according to equation (1.2) when $\gamma_0 = 1.55$ g/cm^3 and when values of α_1 are equal to: 0.001; 0.01; 0.02; 0.03; 0.05; 0.1, respectively. These curves determine the compressibility of the soil as a three-component medium.
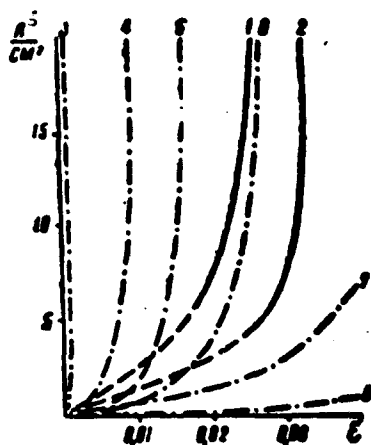

Fig. 4.

The stress in the soil in the direction of the wave movement is equal to the sum of $P_1 = p - \sigma_1$ stresses in the skeleton σ_1 and in the soil as a mixture p. In the case of unsaturated sand the curve, calculated according to equation (1.2), lies much lower than curve 2, $p \ll |\sigma_1|$. One can consider therefore that curve 2, corresponding to the overall compressibility of the soil, practically coincides with the curve determined by the compressibility of the skeleton. The compressibility of the soil as a mixture cannot be taken into account.

With the change in the content of water in the pores not only does the compressibility of the soil change as a mixture, but the compressibility of the skeleton of the soil as well, since the cohesive forces between the solid particles depend on the

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moisture content. As the experiments indicate [3], the effect of
the moisture change on the compressibility of the skeleton is
substantially less than the compressibility of the soil as a mix-
ture. Therefore, in the first approximation one can assume that
curve 2 determines the compressibility of the skeleton $\sigma_1(\epsilon)$ as
well as the water saturated soil. Then, curve 1 corresponds to
the total stress in water saturated soil P_1, curve 2 — to the stress
in the skeleton σ_1, and the difference of these stresses — the
pressure p in the soil as a mixture. When the wave is not a shock
wave, curves 1 and 2 only determine the general character of the
relationship $\sigma_1 = \sigma_1(\epsilon)$ (dashed lines).

The values $p = p_1 + \sigma_1$ are close to the stress determined by
curve 6, which is evidence of the applicability of the equation
of state of a three-component medium (1.2) to the soil as a mix-
ture. Curve 6 corresponds to the same content of air $\alpha_1 = 0.03$
as there would be in a saturated soil.

Let us examine the values of P_1 and p/P_1 determined by curves
1 and 2:

P_1 kgf/cm^2	p kgf/cm^2	p/P_1
20	15.6	0.82
14	10.	0.71
8	4.5	0.56

From the table it is obvious that the ratio p/P_1 with an
increase in the load, increases and approaches unity. This means
that with the larger loads the compressibility of the soil to large
degree is determined by its compressibility as a mixture than with
smaller loads.

From an analysis of the experimental data [3] it follows that
for each soil the value of the load $P_1^* = p^* - \sigma_1^*$, can be shown,
beginning whereby one can ignore the compressibility of the
skeleton and calculate only its compressibility as a three-component

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medium. With an increase in the moisture of the soil, i.e., with a lowering of α_1, the values of P_1^* are reduced.

Approximate values of P_1^* for water saturated soils with various α_1 are given below

α_1	≤ 0.001	0.001-0.01	0.01-0.04
P_1^* kgf/cm^2	0	1-4	4-25

For unsaturated soils the values of P_1^* are equal to several hundreds of atmospheres.

Within the examined range for soils of high and low moisture (Fig. 4) we will have correspondingly:

$$P_1(\varepsilon) = p(\varepsilon) - \sigma_1(\varepsilon), \quad P_1(\varepsilon) = -\sigma_1(\varepsilon). \qquad (2.4)$$

If from the experiment the relationship $P_1 = P_1(\varepsilon)$ is known for a soil of high moisture, then with the help of equations (1.2) and (2.4) we will find the relationship $\sigma_1 = \sigma_1(\varepsilon)$. By knowing from the experiment the relationship $P_1(\varepsilon) = -\sigma_1(\varepsilon)$ for a soil of low moisture, let us find with the help of these same equations the relationship $P_1 = P_1(\varepsilon)$ for the same soil at high moisture.

3. Let us examine the coefficient of lateral pressure. As the experiments indicate, in unsaturated soil when one can take into account only the compressibility of the skeleton, $\varkappa_\tau$ is held practically as a constant value with an increase in the loads up to several tens of atmospheres:

$$\varkappa_\tau = \frac{\sigma_2}{\sigma_1'} = \text{const} < 1. \qquad (2.5)$$

In a water saturated soil when one cannot take into account
the compressibility of the skeleton, with these same loads

$$\kappa_\tau = 1. \qquad (2.6)$$

In a soil where one can take into account both compressibility
mechanisms, the stress in a direction perpendicular to the direction
of the external load, is determined by the equation

$$P_2(t) = p(t) - \kappa_\tau \sigma_1(t) \cdot \qquad (2.7)$$

From this the coefficient of lateral pressure

$$\frac{P_2}{P_1} = \frac{p - \kappa_\tau \sigma_1}{p - \sigma_1}, \qquad (2.8)$$

where κ_τ — coefficient of lateral pressure determined based on the
calculation of the compressibility of the skeleton.

If the external load is not too small, then with its further
increase p increases to a greater degree than does σ_1. From (2.8)
it follows that in this case the ratio P_2/P_1 increases and approaches
unity. The properties of the soil approach those of a liquid.

Let us present the experimental values of P_1 and P_2/P_1 in a
water saturated soil which confirms this deduction:

P_1 kgf/cm^2	7	15	20
P_2/P_1	0.75	0.85	0.90

The average normal stress in the calculation of the two com-
pressibility mechanisms is determined by the equation

$$s = \frac{1}{3}\left(P_1 + P_2 + P_3\right) = p - \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right). \qquad (2.9)$$

In this way a model of the soil was proposed which takes into account the two compressibility mechanisms, and a system of equations was given which determines the planar and spherical movements. In the case of non-unidimensional movements it is also necessary to take into account the two compressibility mechanisms. In this case the behavior of the soil as a mixture is determined according to the equation of state of three-component media, proposed by G. M. Lyakhova [1], and the behavior of the same soil as a solid — with the help of one of the system of equations proposed in [4, 5].

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